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THE VIBRATIONS OF A BOUNDED PLATE IN A FLUID*

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An approximate analytic method is proposed to solve boundary vlaue problems for the harmonic oscillations of a bounded plate in contact with a fluid. The procedure for constructing the successive approximations is such that the first approximation will describe the solution in the main, while subsequent approximations reduce to small refinements on the first approximation. The method is realized in solving a model problem on the plane vibrations of a plate, a strip in a rigid screen with unilateral contact with a liquid medium. Results of a numerical solution of the problem are presented in addition to the approximate analytic solution. By comparing these solutions one can estimate the error of the analytic method in determining the resonance frequencies and modes of plate vibrations in a fluid. This problem has been examined earlier in a somewhat different formulation; the most complete account of the results and bibliographic information can be found in /1/.

1. We will examine the plane problem of the vibrations in cylindrical bending modes of an elastic plate, a strip of width $2l$ and thickness $2h$ $(h/l \ll 1)$ in an infinite rigid screen on the boundary of a liquid half-space $(-\infty \leqslant x \leqslant \infty, z \geqslant 0)$. We will assume the vibrations to be excited by a linearly lumped transverse load $q\delta(x)\exp(-i\omega t)$ applied to the plate at an equal distance from the edges $x = \pm l$ on which we assign hinge-support conditions $w = \partial^2 w/\partial x^2 = 0$ (w is the plate deflection) (Fig.1). After extraction of the time factor $\exp(-i\omega t)$, the system of equations of the joint vibrations of the plate and fluid is written as follows

$$Lw(\xi) + P(\xi, 0) = ql^{-1}\delta(\xi), \quad \Delta P + \gamma^2 P = 0 \tag{1.1}$$
$$\left.\frac{\partial P}{\partial \eta}\right|_{\eta=0,\ |\xi|\leqslant 1} = \omega^2 \rho l w(\xi), \quad \left.\frac{\partial P}{\partial \eta}\right|_{\eta=0,\ |\xi|>1} = 0$$
$$\lim_{r\to\infty} r^{1/2}\left(\frac{\partial P}{\partial r} - i\gamma P\right) = 0, \quad r = (\xi^2 + \eta^2)^{1/2}$$
$$\gamma = kl, \ k = \omega/c, \ L \equiv Dl^{-4} d^4/d\xi^4 - 2\omega^2 \rho_0 h$$
$$\Delta \equiv \frac{\partial^2}{\partial \xi^2} + \frac{\partial^2}{\partial \eta^2}, \quad D = \frac{2Eh^3}{3(1-\nu^2)}, \quad \xi = \frac{x}{l}, \quad \eta = \frac{z}{l}$$

Here $P(\xi, \eta)$ is the acoustic pressure, ρ_0, E, ν are the density, elastic modulus, and Poisson's ratio of the plate material, and ρ and c are the fluid density and the velocity of sound therein.

The wavelengths of the bending waves are considerably less than the wavelengths of space waves in the medium for vibrations of plates in contact with a fluid in the range up to the coincidence frequence /1/. The coincidence frequency is found at the limit of applicability of the dynamic equations of the theory of thin plates; the wavelengths of a free bending half-wave, that exceeds the plate thickness by less than an order, corresponds to it. It can consequently be assumed that the component of the solution corresponding to the spatial waves in the medium should be slowly-varying as compared with the component describing the vibrations of the plate jointly with the fluid in the near-wall layer.

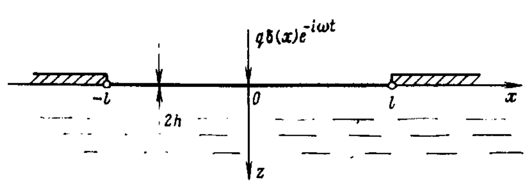

Fig.1

A recursion process to determine the rapidly and slowly varying components of the plate deflection and the acoustic pressure at an arbitrary point of the medium is constructed in /2/ in solving an analogous problem for an unbounded plate. The convergence of the successive

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approximations to the exact solution in a geometric progression with a denominator that is a large parameter is shown. This procedure can be carried over to the problem of the forced vibrations of a bounded plate. Since the first approximation of the process mentioned for the rapidly varying component of the solution describes the near pressure field of the non-radiating surface-wave type, corresponding formulas will yield unlimited growth of the vibration amplitudes at resonance frequencies of the system. The latter construction of the slowly-varying components of the solution satisfying the radiation condition does not alter the situation since this component is added with the first approximation. Consequently, the procedure for constructing the first approximation in this problem should be corrected so that the damping influence of the sound radiation is already be taken into account at this stage of obtaining the rapidly-varying component.

Following the computations in /2/, we will represent the rapidly varying component of the solution of system (1.1) for vibration modes symmetrical about the line of load application in the form

$$w_1(\xi)=\sum_{j=1}^{3} f_j(\xi),\quad P_1(\xi,\eta)=-\omega^2\rho l\sum_{j=1}^{3}\alpha_j^{-1}f_j(\xi)\exp(-\alpha_j\eta) \tag{1.2}$$
$$f_j(\xi)=c_j\cos\beta_j|\xi|+b_j\sin\beta_j|\xi|,\quad \beta_j=(\alpha_j^2+\gamma^2)^{1/2}$$

Here $\alpha_1>0$, $\operatorname{Re}\alpha_{2,3}>0$ are the roots of the characteristic equation, corresponding to exponentially damped integrals of the acoustic pressure in the neighbourhood of the plate

$$\alpha^5+2\gamma^2\alpha^3-\gamma^2(g_1-\gamma^2)\alpha-g_0\gamma^2=0 \tag{1.3}$$
$$g_1=3(1-\nu^2)\left(\frac{l}{h}\frac{c}{c_0}\right)^3,\quad g_0=\frac{l}{2h}\frac{\rho}{\rho_0}g_1,\quad c_0=\left(\frac{E}{\rho_0}\right)^{1/2}$$

For an antisymmetric exciting load, the vibration modes have an analogous form with the addition of the multiplier $\operatorname{sgn}\xi$.

Six conditions must be formulated to determine the six unknown constant c_j, b_j $(j=1,2,3)$ in (1.2). Four are known, two boundary conditions and two conditions extracting the singularity of the plate deflection on the force line of action that agrees with the principal singularity of the deflection function for plate vibrations in a vacuum. The non-penetration condition also imposes a constraint on the singularity of the pressure function by relating it to the singularity of the deflection function on the force line of action. The remaining arbitrariness in the constants is reduced when giving the values of the pressure function on the plate edges.

To derive this condition we will represent the exact value of the fluid pressure on the plate surface in the form of an integral of the product of the deflection function and Green's function of the Neumann problem for the Helmholtz equation

$$P(\xi,0)\equiv w*G=\omega^2\rho l\int_{-1}^{1} w(\xi_0)G(\xi,0;\xi_0,0)\,d\xi_0 \tag{1.4}$$
$$G(\xi,\eta;\xi_0,\eta_0)=-\frac{i}{4}\sum_{k=1}^{2}H_0^{(1)}(\gamma[(\xi-\xi_0)^2+(\eta-(-1)^k\eta_0)^2]^{1/2})$$

($H_0^{(1)}(x)$ is the Hankel function of the first kind of zeroth order). Substitution of this expression into (1.1) results in an integrodifferential equation in the plate deflection. The first approximation of the deflection function (1.2) satisfies this equation to the accuracy of the difference between $P_1(\xi,0)$ and w_1*G. This difference can be made zero in not more than two points (for the symmetric solution) by using the remaining arbitrariness in the selection of the constants. The best approximation is obtained in the case when these points are chosen on the plate edges

$$P_1(\xi,0)-w_1*G=0,\quad \xi=\pm1$$

Substitution of (1.2) into the condition listed above results in six algebraic equations to determine the constants c_j, b_j $(j=1,2,3)$

$$\sum_{j=1}^{3}x_{jk}=-\frac{\delta_{k2}ql^3}{2D},\quad k=1,2,3;\quad \sum_{j=1}f_j(1)\beta_j^n=0,\quad n=0,2 \tag{1.5}$$
$$\sum_{j=1}^{3}\alpha_j^{-1}f_j(1)=\frac{i}{2}\sum_{j=1}^{3}\int_{-1}^{1}f_j(\xi)H_0^{(1)}(\gamma(1-\xi))\,d\xi$$
$$x_{j1}=\beta_jb_j,\quad x_{j2}=x_{j1}\beta_j^2,\quad x_{j3}=x_{j1}\alpha_j^{-1}$$

The solution w_1, P_1 constructed in this manner can be supplemented to the exact by introducing new unknown functions w_2, P_2, where $w=w_1+w_2$ and $P=P_1+P_2$.

The equation for the second approximation of the function w_2 is homogeneous while the equation for P_2 is inhomogeneous with the right side

$$2\omega^2\rho l\delta(\xi)\sum_{j=1}^{3}x_{j1}\exp(-\alpha_j\eta)$$

reflecting the discontinuity in the derivative of the first approximation (1.2) for the pressure function on the force line of action. Because of the exponential damping of the coefficient for the delta function, this discontinuity is localized on a small portion of the z-axis near the plate.

The solution of the inhomogeneous equation for P_2 will be constructed by using Green's function of the Helmholtz equation that satisfies the homogeneous Neumann condition on the plate surface and the radiation condition at infinity. Its convolution with the right side of this equation determines the radiation component of the pressure which can be represented in the form of a sum $P_{2r}=P_2^{(\infty)}+P_2^{(1)}$, where $P_2^{(\infty)}$ is in agreement with the pressure produced in vibrations of an infinite plate subjected to a linearly lumped force /2/, while $P_2^{(1)}$ characterizes the sound radiation from the plate edges.

On the plate surface, P_{2r} does not vanish identically and does not satisfy the first equation of (1.1). We reduce the residual which is formed by setting

$$P_2=P_{2r}+P_{20},\ \partial P_{20}/\partial\eta\,|_{\eta=0}=\omega^2\rho l w_{20}$$

Substituting this sum into the system (1.1) we obtain a homogeneous Helmholtz equation in P_{20} and in the equation

$$Lw_{20}+P_{20}(\xi,0)=-P_{2r}(\xi,0) \tag{1.6}$$

It is different from the first equation in (1.1) by its right side. Consequently, (1.2) can be used for w_1 as an approximate Green's function of Eq.(1.6). Then

$$w_{20}(\xi)=-\frac{l}{q}\int_{-1}^{1}w_1(\xi-\xi_0)P_{2r}(\xi_0,\ 0)\,d\xi_0$$

The functions w_2, P_2 determined in this manner differ from the exact solution by the quantities w_3, P_3. Consequently, the recursion process can be continued: by writing the sums $w=w_1+w_2+w_3$, and $P=P_1+P_2+P_3$ and substituting them into the initial system of equations we obtain the third approximation equation.

2. Let us compare the approximate analytic solution with the results of a numerical solution. We construct the numerical solution of the problem under consideration on the basis of an integrodifferential equation which is written as follows in dimensionless form

$$\frac{d^4w^\circ}{d\xi^4}-p_1\lambda^2w^\circ=\frac{i}{2}p_2\lambda^2\int_{-1}^{1}H_0^{(1)}(\gamma r)\,w^\circ(\zeta)\,d\zeta+p_3q^\circ\delta(\xi-\xi_q) \tag{2.1}$$

$$r=|\xi-\zeta|,\quad \gamma=p_4\lambda,\quad \lambda=2\omega l\left[\frac{\rho_0(1-\nu^2)}{E}\right]^{1/2}$$

$$w^\circ=\frac{w}{\overline{W}},\quad q^\circ=\frac{q}{\overline{F}}$$

$$p_1=12\left(\frac{l}{h}\right)^2,\quad p_2=p_1\frac{l}{h}\frac{\rho}{\rho_0},\quad p_3=8\frac{l^3\overline{F}}{D\overline{W}}$$

$$p_4=\frac{1}{c}\left[\frac{E}{\rho_0(1-\nu^2)}\right]^{1/2}$$

Here λ is the dimensionless frequency, $p_j\ (j=1,2,3,4)$ are dimensionless parameters, ξ_q is a coordinate of the line of load application, and $\overline{W}$, $\overline{F}$ are scaling constants.

Extracting the irregular component in the solution, we represent the deflection function in the form (we henceforth omit the superscript $^\circ$)

$$w(\xi)=w_0(\xi)+p_3q\,G_0(\xi,\xi_q)$$

where $G_0(\xi,\xi_q)$ is Green's function of the problem without taking account of the fluid. The regular component $w_0(\xi)$ satisfies an equation of the form (2.1) in which the second term on the right side of the equation is

$$\frac{i}{2}p_2p_3\lambda^2qH(\xi,\xi_q),\quad H(\xi,\xi_q)=\int_{-1}^{1}H_0^{(1)}(\gamma r)\,G_0(\zeta,\xi_q)\,d\zeta$$

For a numerical solution of the integrodifferential equation for $w_0(\xi)$ we will introduce in the section $-1\leqslant\xi\leqslant1$ a uniform net

$$\Delta_{N+1}=\{\xi_j=-1+(j-1)\delta,\ j=1,2,\ldots,N+1;\ \delta=2/N\}$$

Let us approximate the equation at the nodes $j=2,3,\ldots,N$ by replacing the derivative of the function $w_0(\xi)$ by a difference ratio of the second order of accuracy, and the function $w_0(\xi)$ under the integral sign by a piecewise-linear interpolating function (in the mesh Δ_{N+1}).

We introduce the points outside the contour $\xi_{N+2} = \xi_{N+1} + \delta$, $\xi_0 = \xi_1 - \delta$ to approximate the derivative in (2.1) at the points $j = 2, N$, and the derivatives in the boundary conditions. Then by using equations approximating the boundary conditions, values of the mesh function w_j at the nodes $j = 0, 1, N+1, N+2$ are eliminated from the resolving system of algebraic equations. We consequently arrive at equations that are written as follows in matrix-vector form:

$$\mathbf{A}_p\mathbf{W} = (i/2)\, p_2\lambda^2\delta^4\, (\mathbf{A}_g\mathbf{W} + p_3 q\mathbf{H}) \qquad (2.2)$$
$$\mathbf{W} = [w_2, w_3, \ldots, w_N]^T, \quad \mathbf{H} = [H_2, H_3, \ldots, H_N]^T, \quad H_j = H(\xi_j, \xi_q)$$

Here $\mathbf{A}_p$, $\mathbf{A}_g$ are matrices of order $N-1$, approximating the differential and integral operators. The tape matrix $\mathbf{A}_p$ contains real elements. The complex elements of the matrix $\mathbf{A}_g$ are expressed by the formula

$$a_{jk} = -(k-j-1)\, h^0_{k,j+1} + (k-j+1)\, h^0_{k+1,j+1} + \delta^{-1}(h^1_{k+1,j+1} - h^1_{k,j+1})$$
$$j, k = 1, 2, \ldots, N-1$$

The matrices $\{h_{jk}{}^0\}$, $\{h_{jk}{}^1\}$ $(j = 1, 2, \ldots, N;\ k = 1, 2, \ldots, N+1)$ have the following simple structure

$$\{h^0_{jk}\} = \begin{Vmatrix} a_1 & a_1 & a_2 & \ldots & a_N \\ a_2 & a_1 & a_1 & \ldots & a_{N-1} \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ a_N & a_{N-1} & a_{N-2} & \ldots & a_1 \end{Vmatrix}, \quad a_j = \int\limits_{(j-1)\delta}^{j\delta} H_0^{(1)}(\gamma\xi)\, d\xi$$

$$\{h^1_{jk}\} = \begin{Vmatrix} -b_1 & b_1 & b_2 & \ldots & b_N \\ -b_2 & -b_1 & b_1 & \ldots & b_{N-1} \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ -b_N & -b_{N-1} & -b_{N-2} & \ldots & b_1 \end{Vmatrix}$$

$$b_j = \int\limits_{(j-1)\delta}^{j\delta} H_0^{(1)}(\gamma\xi)\, \xi\, d\xi$$

After separating real and imaginary parts in (2.2) we obtain a system of algebraic equations of order $2(N-1)$ in the real and imaginary parts of the values w_j that approximate the regular component of the deflection $w_0(\xi)$ at the nodes $j = 2, 3, \ldots, N$.

The case of a steal plate making contact with water was examined in the numerical investigation made with the algorithm mentioned. The following values were given for the parameters of the problem: $l/h = 100$, $p_1 = 1.2\cdot10^5$, $p_2 = 1.538\cdot10^6$, $p_3 = 480$ and $p_4 = 3.494$. The value of the deflection of a plate under static loading by a lumped force applied at the middle point was taken as the scaling constant $\overline{W}$: $\overline{W} = ql^3/(6D)$.

The results of the numerical investigation were compared with the results of the approximate analytic method (Sect.1), and in the case of resonance vibrations, with the solution of the problem of free vibrations of an infinite plate $(-\infty \leqslant x \leqslant \infty)$ that is in contact with a fluid. In the latter case a solution of the standing-wave type exists $w(\xi) = w_0 \sin k_0\xi$. The dispersion equation that is a condition for this solution to exist reduces to (1.3) for the quantity $(k_0{}^2 - \gamma^2)^{1/2}$, $k_0 > \gamma$.

We present below, in the first row, five values of the first resonance frequencies λ_n found by the numerical method. The second row contains the results of a computation using (1.2)-(1.5). The estimates of the resonance frequencies determined from the dispersion equation for an integer number of half-waves covering the width of the bounded plate are presented in the third row

1)	0,0062	0,0584	0,151	0,301	0,504
2)	0,0068	0,0618	0,154	0,299	0,509
3)	0,0126	0,0653	0,167	0,320	0,528

The solution the standing-wave type can be obtained from (1.2) if $P_1(\pm1, 0) = 0$ are taken as boundary conditions for the pressure function. In this case the resonance vibration modes agree in form with the natural vibrations modes of a bounded plate in a vacuum. Therefore, values of the resonance frequencies can be determined to a first approximation by using the above-mentioned dispersion equation, however such an estimate is quite rough for the lowest frequencies. We note that the estimate of the first resonance frequency by the Lamb method /3/ has an error of the same order.

The modulus of the complex deflection amplitude $w = w_R + iw_I$ reaches its maximum value for resonance vibrations, where $\max_\xi |w_I(\xi)| \gg \max_\xi |w_R(\xi)|$. As computations show, graphs of the functions $w_I(\xi)$ for resonance vibrations in the first symmetric and antisymmetric modes do not differ, in practice, from graphs of the natural modes of the deflection of a bounded plate in a vacuum. However, noticeable differences are observed for the second and subsequent symmetric and antisymmetric modes. Graphs of the functions $w_I(\xi)$ are represented by the solid lines in Figs.2 and 3 for resonance vibrations in the second and third symmetric modes (curve *1* is the numerical solution, and curve *2* is the approximate analytic solution) and corresponding natural vibrations modes of a hinge-supported plate in a vacuum, normalized to the maximum value of w_I, are superposed by dashes. It is seen that the approximate analytic solution (1.2)-(1.5) enables us to determine the resonance vibration frequency not only with sufficient accuracy, but also describes correctly the singularities of the resonance vibration modes of a plate in a fluid, which distinguish them from the corresponding vibration modes in a vacuum. Despite the fact that these differences are not very great (the discrepancy in the half-wave amplitudes is 27% for the second symmetric mode and 16.5% for the third), they exert it considerable influence on the value of the deflection amplitudes and the resonance acoustic field generated by the plate.

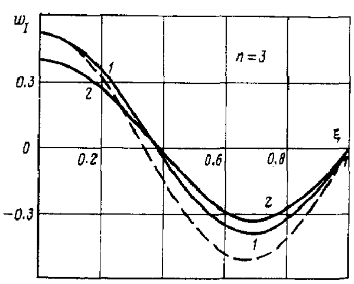

Fig.2

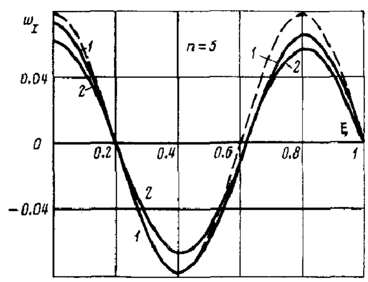

Fig.3

For a hinge-supported plate in a vacuum, the n-th natural symmetric vibration mode is described by the function $w_n = A_n \cos(\pi n\xi/2)$ $(n = 1, 3, 5, \ldots)$. Considering that it also describes the resonance vibrations in a fluid, as is assumed in a number of papers, we will determine the amplitude factor A_n' at the corresponding resonance frequency. To do this, we substitute w_n into the integrodifferential Eq.(2.1), multiply its left and right sides by $\cos(\pi n\xi_0/2)$ and integrate over the width of the plate. We hence arrive at the expression

$$A_n = \frac{q}{lW}\left[D\left(\frac{\pi n}{2}\right)^4 - 2\omega^2\rho_0 h - \omega^2\rho l\,\frac{i}{2}\,I_n\right]^{-1}$$

$$I_n = \int_{-1}^{1}\int_{-1}^{1} \cos\frac{\pi n\xi}{2}\cos\frac{\pi n\xi_0}{2}\,H_0^{(1)}(\gamma\,|\,\xi - \xi_0\,|)\,d\xi_0\,d\xi$$

The real part of the expression in the square brackets vanishes at the resonance frequency, while the imaginary part determines the resonance amplitude. Computations executed by means of this formula yield the following values of the dimensionless amplitude for the resonance vibrations of a plate in contact with a fluid in the first, second, and third symmetric modes: $|A_1| = 3.3$; $|A_3| = 0.065$; $|A_5| = 0.0012$. The corresponding maximum amplitudes obtained by the numerical method are 3.9; 0.52; 0.078. The approximate analytic solution (1.2)-(1.5) yields the values 4.4; 0.40; 0.065.

A comparison of these results shows that, with the exception of the first resonance mode, the utilization of the natural plate vibrations modes in a vacuum to describe the resonance hydroelastic vibrations of a plate, that are symmetrical about the central plane $x = 0$, result in a substantial reduction in the resonance vibration amplitudes. This is explained by the compensating influence of the fluid, whereupon the integral of the resonance deflection function taken over the width of the plate is much less than the integral of the function describing the natural mode of symmetric plate vibrations with the same number of waves, in a vacuum. The error in determining the deflection amplitudes for resonance vibrations involves an error in calculating the pressure in the far field. Thus, for instance, identifying the second symmetric resonance mode of hydroelastic plate vibrations (with three half-waves over the width) with its natural mode results in an eightfold reduction in the maximum deflection amplitude and in a $\sqrt{8}$-fold reduction in the far-field pressure.

On the other hand, the compensating influence of the fluid for the antisymmetric plate vibration modes is felt in a reduction in the resonance vibration amplitudes as compared with their expected values when using the corresponding natural vibration modes in a vacuum. Nevertheless, because of the general equilibrium with respect to the non-deformed state, their level as a whole is considerably higher than for the neighbouring symmetric modes. This is manifested most clearly in a comparison of the maximum amplitudes of the deflection for the first symmetric and antisymmetric modes: 3.9 for $\lambda_1 = 0.0062$ against 33 for $\lambda_2 = 0.0584$ $(\xi_q = 0.4)$. It is seen that despite the general tendency towards a reduction in the resonance vibration amplitudes as the frequency increases, the amplitude of the first antisymmetric mode is almost an order of magnitude higher than for the first symmetric vibration mode.

Therefore, the influence of the fluid on the vibrations of a bounded plate manifests itself not only in a reduction in the resonance frequencies, but also in the distortion of the resonance plate vibration modes that exerts a substantial influence on the deflection amplitude and the acoustic pressure in the medium.

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HIGH-FREQUENCY LONGITUDINAL VIBRATIONS OF ELASTIC RODS*

LE KHAN' CHAU

One-dimensional equations are constructed for the high-frequency longitudinal vibrations of elastic rods. Problems in a section are formulated to determine the "effective" elastic characteristics of the rod. In the case of a circular rod, the elastic characteristics, dispersion curve, and spectrum are found. Comparisons are made with analogous results of the three-dimensional theory of elasticity and with experiment.

1. One dimensional theory of high-frequency longitudinal vibrations of rods.

We consider an isotropic homogeneous straight rod of length $2L$ with a constant cross-section S that occupies a volume V in the non-deformed state in the Cartesian coordinate system $x^1, x^2, x^3 \equiv x$ (the superscript 3 is usually omitted). We place the origin of the coordinate system at the centre of the rod and direct the x axis along its central axis. We will assume the cross-section to be centrally symmetric (if $(x^1, x^2) \in S$, then $(-x^1, -x^2) \in S$).

Under given initial conditions the rod performs vibrational motion. The problem is to construct a one-dimensional dynamic model of the rod high-frequency vibrations that is asymptotically exact in the long-wave domain, and is moreover qualitatively descriptive of the rod integral characteristics in the short-wave domain. Taking a variational approach as a basis /1, 2/, we postulate that the rod motion will occur in conformity with the following variational principle

$$\delta \int_{t_0}^{t_1} \int_{-L}^{L} \Lambda \, dx \, dt = 0, \quad \Lambda = K - \Phi \tag{1.1}$$

where K and Φ are the one-dimensional kinetic and internal energy densities of the rod.

The formulas

$$K = {}^1/_2 \rho u_{,t}^2, \quad \Phi = {}^1/_2 E u_{,x}^2 \tag{1.2}$$

turn out to be true in the classical theory of longitudinal vibrations of a rod, where u is the longitudinal displacement averaged over the cross-section, E is Young's modulus, and ρ is the density of the elastic material of the rod. The model (1.1), (1.2) describes the low-frequency, long-wave vibrations of the rod. It is natural to assume that as the vibration frequency increases the internal degrees of freedom that characterise the new modes (branches) of the rod vibrations will become substantial and these vibrations can be described, in a certain frequency range, by eliminating an appropriate set of internal degrees of freedom in the number of arguments of the functions K and Φ. Within the framework of this approach it is most important to determine the set of essential degrees of freedom and to set up the

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